Taking the Severe out of Perseverance:

Strategies for Building Mathematical Determination

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Abstract

Perseverance in mathematics is a multifaceted construct involving students’ interests and proclivities, their will and skill. Often it is portrayed as a kind of trait that a student possesses—a kind of generalized intellectual toughness—rather than as a behavior that emerges from a variety of subjective appraisals that students make in particular contexts and situations. We propose a model of the development of perseverance in mathematics that redefines perseverance as not only overcoming obstacles en route to achieving a goal, but as a self-regulatory strategy that consciously redefines the obstacle in terms of both its conceptual and motivational characteristics. Mathematical perseverance, then, is seen less as a trait and more as an interplay between mathematical tasks, mathematics as an intellectual pursuit, and the goals, interests, and resources students bring to the learning environment. Following a section that presents our definition and model of perseverance, we discuss four central aspects of perseverance—interests and identity, setting goals, utilizing resources, and anticipating consequences. In each of these four sections, we offer suggestions for how teachers can use insights from the existing research to improve student perseverance in mathematics.
If your determination is fixed, I do not counsel you to despair. Few things are impossible to diligence and skill. Great works are performed not by strength, but perseverance.

-Samuel Johnson

Hard work and perseverance enjoy almost mythical status in the American mindset. They are seen as ways to overcome personal limitations on the one hand, and as social virtues on the other (Weber, 2002). Yet one of the most perplexing and frustrating problems facing US K-12 education is fostering students’ mathematical perseverance. To develop a competent and innovative workforce, students must come to recognize that mathematics might be interesting, important, and useful to them as they pursue their ambitions—yet this remains a relatively rare disposition among students in contemporary mathematics education.

Falling out of love with mathematics may not be particularly problematic in the immediate term. Currently in the United States, a person can obtain a good job, support a family, and live a comfortable life with only a background in high school or early collegiate mathematics. But as the competition for innovative, technically skilled knowledge workers increases, the demand for more mathematically sophisticated thinkers increases accordingly. Whereas culminating one’s mathematical experiences in grade 11 or 12 with Algebra II was heretofore acceptable, current knowledge workers need conceptual tools from calculus, statistics, and discrete mathematics—typical college mathematics content—to be able to perform effectively at the most basic skilled jobs, as affirmed in the Common Core State Standards Initiative (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010). This implies that students need to either: (1) take additional years of mathematics in college, trade school, or some open certification program; (2) accelerate their mathematical experiences in K-12; or (3) for the technical workforce, both.

Given this reality, requiring increased investment of time, more intense effort, and more challenging curriculum seems to be a generally positive policy move for the US educational system. However, these enhanced challenges likely present severe obstacles for students with low mathematical motivation. Clearly, redesigning learning environments and practices is necessary to help more students face these challenges, overcome the cognitive and social obstacles that prevent them from succeeding, and perhaps even enjoy the ride along the way. This is the central focus of this paper.

Defining Perseverance

To untangle students’ mathematical perseverance as a construct that develops in the long-term, we need to provide clarity on two constructs currently used synonymously in the literature: persistence and perseverance. The Oxford English Dictionary defines persistence as a “firm or obstinate continuance in a course of action in spite of difficulty or opposition” and perseverance as a “steadfastness in doing
something despite difficulty or delay in achieving success.” Aside from the slight semantic difference between “steadfastness” and “obstinate continuance,” the two terms appear to be functionally equivalent. However, this is not the case psychologically.

In working towards any goal, an individual may anticipate challenges and obstacles at the start. The anticipated obstacles become part of the learner’s plan of attack for achieving the goal, but he or she may not anticipate all obstacles. When confronted with the unanticipated obstacles that inevitably arise, the learner faces three choices. The learner may choose to (1) stick with the status quo (the loose mental plan of attack formed at the outset), (2) choose to alter his or her mental plan and seek out additional resources needed to achieve the goal, or (3) quit the current plan and abandon pursuit of the goal.

Sticking with the status quo, as we have described it, defines persistence (Peterson & Seligman, 2004). In the face of an unanticipated obstacle, the individual may continue in the same vein in which he or she was already working, even if that vein may be the root cause of the unanticipated obstacle. For example, a child who continues to use her repeated subtraction strategy for solving division problems will be unsuccessful when trying to solve division by fractions. With no other strategy to employ, she may continue to try repeated subtraction because it has worked in the past for whole numbers, even when it continues to prove ineffective or impossible for division by fractions. When the learner chooses to alter the plan so as to continue pursuing the goal, however, we say that he or she shows perseverance. Here the learner continues to seek the fulfillment of some goal, but makes the conscious choice to alter or adapt strategies and approaches. Making this distinction allows us to classify learning behaviors into productive and non-productive types. For example, perseverating, obsessively trying the same strategy over and over again to no avail (Jowker-Baniani & Schmuckler, 2014), is non-productive persistence. Cognitive flexibility, which is the ability to strategize and recruit resources to meet problem demands (Maddox & Markman, 2010; Spiro, Vispoel, Schmitz, Samarapungavan, & Boerger, 1987), and self-regulation, which is the control of one’s behavior towards some goal (Carver & Scheier, 2001), are both closely linked to perseverance.

It quickly becomes apparent that perseverance necessitates persistence. Being able to stick with a strategy until it proves non-productive is a necessary condition for productive problem solving. But perseverance additionally requires some thoroughness and diligence in performing a task, for example, by altering strategies to continually work towards the end goal. It also requires finding some means for evaluating the goodness of a strategy—understanding what constitutes good mathematical thinking, appropriate procedures, and effective verification practices. With this combination of persistence and conscientiousness in mind, we offer the following definition of perseverance:

Perseverance is the continuance of effort, carried out in a thorough and diligent manner, towards some perceived goal while overcoming difficulties, obstacles, or
discouragement along the way by amending one’s plan of attack.

Perseverance, therefore, as we have defined it, is not a trait. Instead, it can be seen as a pattern of relatively consistent decisions over time to continue to work hard towards a goal. In our framework, learners employ their interests and identities to choose activities and pursuits in which to engage; they create and evaluate their efforts in terms of current and future goals; they utilize psychological, social, and material resources to enhance their potential for success; and they weigh the consequences of their actions to determine how to act in the future. What is important here is that these aspects of perseverance (interests and identities, goals, resources, and consequences) are malleable factors, meaning that teachers, parents, and the student herself can influence them in an attempt to increase the likelihood that she will persevere mathematically. In the following sections, we discuss these factors, showing how each contributes to mathematical perseverance. We follow up each discussion with specific suggestions and strategies that educators can effectively employ to encourage mathematical perseverance.

**Four Aspects of Perseverance**

**Interest and Identity**

In this section, we show that learners’ interests and identities significantly determine the courses their lives will take. Personal interest and identity help students choose the tasks to which they will apply their effort as well as the level of effort they are willing to expend, orienting their behavior in personally productive directions. At the end of this section, we provide some suggestions regarding how teachers can utilize this information to help students make choices that help them persevere through challenge and develop self-concepts that incorporate mathematics as both interesting and useful.

**Situational interest.** Most K-12 students do not enroll in mathematics because they like the content. Students generally engage in mathematical tasks because it is expected and because others around them are doing so, though they expend different amounts of effort depending on the degree to which they find the situation interesting. The interestingness of any task, as a student engages in it, varies over time and across different tasks. Such interest is called “situational interest,” and it contrasts with the more stable long-term aspirations and pursuits a learner gravitates towards in his or her lifetime, which are called “individual interests.”

Research shows that students who by-and-large have positive mathematics experiences—i.e., who regularly experience situational interest—tend to enjoy mathematics more and tend to seek out more mathematically sophisticated pursuits in the long-term. Moreover, by tailoring the context within which mathematics tasks are situated to the long-term, individual interests of learners, we can lead learners to associate their individual interests with the mathematics they are engaged in currently, thereby fostering situational interest. For example, a student who is interested in astronomy may become intrigued by the
mathematics of scientific notation when contemplating the size and age of our galaxy because scientific notation is critical for understanding the vast distances and timescales of our universe. The literature consistently documents the positive effects of individual interest on engagement, persistence, and preference for challenge in academic tasks and subsequent career choices (Deci & Ryan, 2002; Eccles & Wigfield, 2002; Betz & Hackett, 2006; Hackett & Betz, 1989; Efklides & Petkaki, 2005; Ainley, Hidi, & Berndorff, 2002; Köller, Baumert, & Schnabel, 2001; Laukenmann, Bleicher, Fuß, Gläser-Zikuda, Mayring, & von Rhöneck, 2003; Linnenbrink & Pintrich, 2004). Positive emotions associated with learning in an interest area become associated with the mathematical concepts and skills being learned—students feel better and happier when studying their interests. Over time, these situational interests become generalized into a stable value system that may include mathematics.¹

Now what happens when the learner has constructed no clear set of interests related to mathematics? In such cases, learners may determine whether or not a mathematical task is potentially interesting by comparing its level of challenge to their own mathematical ability. If the challenge is not too much (or too little), and if a student’s ability is up to the challenge, the student can tentatively label the task as “kind of interesting.” Continued engagement in such tasks allows them to be associated together as “interesting” topics, and over time into mathematics as an individual interest (Middleton & Toluk, 1999; see also Hidi & Renninger, 2006; Krapp, 2002; Middleton, 2013). Research shows that students are able to actively control their ongoing effort in tasks that are interesting to them. They can generate strategies to make un-interesting tasks more interesting if they have a reason to value the task, and such strategies can be taught to them (Sansone, Wiebe, & Morgan, 1999). The trick to engendering interest where it doesn’t occur naturally is finding that “sweet spot” where the mathematics very closely approximates learners’ knowledge and capabilities in the moment.

Importantly, students who report interest in mathematics also tend to report enjoyment of and pride in mathematics (Tulis & Ainley, 2011). Interest and its associated emotional states mediate students’ self-regulation strategies, keeping them engaged emotionally even in the face of failure, making it more likely that they will persevere in the mathematical tasks than if they did not experience interest. As such, the development of mathematical interest should become a central goal of mathematics curriculum and instruction.

Identity. When people make statements about their interests, they are making statements about themselves. You might say, “I am interested in jazz.” In doing so, you connect yourself to a whole slew of experiences—situational interests—that together form your interest in a particular music genre. You also identify how you spend your time, and you identify a community of others (i.e., jazz aficionados) with

¹ Note also that social interests are part of students’ larger set of values and should not be ignored in the design of curriculum and classroom management.
whom you share common experiences, language, and worldview. These configurations of experiences, others, language, and worldview form our identities—how we see ourselves in relation to others in the world. They situate us in a community and provide support and structure for our learning behavior in that subject area (see Wenger, 1998). Unfortunately for mathematics, research has shown that students begin to de-identify with mathematics, typically around the middle grades (Eccles, Wigfield, Midgely, Reuman, Iver, & Feldlaufer, 1993; Kloosterman & Gorman, 1990).

We can think of the development of mathematical identity as being the stories learners create about themselves that describe their memberships in or exclusions from various communities (Sfard & Prusak, 2005; see also Cobb, Gresalfi, & Hodge, 2009). Students compare themselves against the norms of mathematical competence in their class(es), construct sets of “stories” that define their own proclivities and handicaps, and use these stories to help them decide when and to what extent they will engage in the social activity of doing mathematics. A student who has developed a positive sense of competence in relation to her class and who has developed individual interest in mathematics will readily engage in challenging mathematical tasks, but will continue to monitor her situational interest to determine whether or not continued engagement is worthwhile. Together, these identity-related beliefs and personal interests direct and channel a learner’s tendency to persevere.

**What teachers can do.** To capitalize on the research on interest and identity, teachers need to keep three things in mind: (1) know your students’ individual interests; (2) choose or develop tasks that have a high degree of challenge and varying degrees of control; and (3) emphasize mathematical norms that promote sharing one’s ways of thinking *even if they may not be correct*. Regarding the first point, the more you know your students’ individual interests, the more you are able to choose tasks and contexts to “hook” their interest and help them to see mathematics as useful and associated with the pursuits they are passionate about. Of course, not all tasks are amenable to all interests, so an appropriate variety of contexts is necessary to engage each student reasonably over the course of a year.

Whether or not tasks are situated within students’ interests (but especially if they are not), it is critical to gauge the level of challenge tasks afford relative to students’ abilities. High challenge is generally desirable for most students. But providing supports by selecting appropriate tasks, forms of representation, tools, and other material resources is critical to empowering students and ensuring that the level of challenge is appropriate for their abilities. Additionally, in mathematics, heterogeneous grouping of students has been shown to be effective in improving the performance of struggling students without sacrificing that of more able students (Burris, Heubert, & Levin, 2006; Linchevski & Kutscher, 1998).²

Lastly, the norms for mathematical competence are the background upon which students build

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² The caveat here is that strategies for effective groupwork in heterogeneous classrooms differ from those that work well in homogeneous classrooms. See Boaler (2006) for an excellent exposition of effective groupwork strategies in heterogeneous mathematics classes.
their identities, and norms can channel students’ behaviors in productive ways. Scardamalia and Bereiter (2003) provide a detailed theory of “knowledge building” communities that emphasize the norm that each person in the classroom is responsible for the improvement of the knowledge of the class. When ideas are poorly understood, the student who recognizes her own difficulties has a responsibility to make her struggle public so that the class can improve her knowledge. In doing so, she helps both herself and the class as a whole. Likewise, when a student perceives that she has an important insight, it is imperative that she share her understanding to improve the knowledge of her peers. In such a classroom, being wrong is not considered a deficit, but an opportunity to learn, and being correct is an impetus to help. The development of mathematical identity in such classes has tremendous potential to promote perseverance because adaptive persistence, as we have defined perseverance, is essential to knowledge improvement.

The following section deals with how the goals students create impact their engagement in both the short-term and long-term. With goals, as with the development of interest and identity, what we do mathematically in the short-term is critical to the development of perseverance over the long haul.

**Development of Goals**

In the mathematics classes we took, we were introduced to what needed to be accomplished to learn mathematical concepts and skills, but more often than not, our teachers failed to help us understand why such concepts and skills should be learned and to what ends they could be applied. This made it difficult to see how our goals for learning mathematics related to the larger set of goals we had for living our lives and for building a better future for ourselves. In this section, we describe the nature of goals, focusing on learning goals versus performance-related goals; the relevance of goals to one’s immediate needs versus future plans; and the impact of different types of goals on perseverance. Following this discussion, we examine strategies for improving perseverance through effective goal setting and monitoring.

The goals we set for ourselves define a state of being we desire to enter—an end state representing the culmination of a complex chain of behaviors over time. In essence, they are a projection of a possible future we might shoot for. As an end state, a goal can be as simple as obtaining the intercept of a line in a homework problem. It could also entail an affective state like a sense of achievement, or a state as long-term and complex as the adoption of a new identity. We stress here that any mathematical experience, if goal-directed, will involve an end state that is at once cognitive (i.e., obtaining a new understanding of the problem and/or of the underlying mathematics) and affective/motivational (i.e., experiencing satisfaction or avoiding feelings of failure) (see Pintrich, 2000a). Perseverance can be thought of as one’s progress, over time, in moving from a current state to a desired state, all the while dealing with the obstacles and opportunities that arise along the way.

Goals vary along four basic dimensions: (1) specificity; (2) proximity to the learner’s current
state; (3) focus on learning versus performing; and (4) approach versus avoidance of an end state. We discuss each of these dimensions in turn, stressing that any goal will display a bit of each characteristic to at least some degree.

**Goal specificity.** As with interests, the perceived challenge of a task will determine the kinds of goals an individual will develop. In other words, the goals a learner will set depend upon the ways in which the learner views herself in relation to the demands of the task. If the learner holds a negative view of her mathematics ability (i.e., has a poor mathematical identity), then the goal of getting an “A” on an assignment may be perceived as unattainable. However, if the student were to view herself as being proficient at mathematics, then the goal of getting an “A” on the same assignment may now be perceived as relatively easy. These self-held perceptions of ability influence students’ behavior—if they have negative views of their ability, they may simply find ways to get by rather than trying to excel, but if they view their ability positively, they may seek to maximize learning as they work toward achieving the goal.

To help students make appropriate evaluations of their abilities, it is helpful to work with them to set specific goals that focus on a particular task and to articulate the ways in which learners will evaluate their achievement of the goal (Ford, 1992; Latham & Locke, 1991; Harackiewicz & Sansone, 1991). In general, specific end goals promote perseverance because they allow the learner to construct a specific (and therefore manageable) strategy for goal resolution. Vague goals like “you will need this mathematics for the future” do not provide the criteria by which students can effectively assess their progress and determine when the goal has been accomplished. Questions like, “What about the future? What kinds of professions use this mathematics? How is it used? What specifically about this task can you put in your ‘pocket’ to remember later?” are potentially effective means of helping students put their mathematics learning experiences in proper perspective.

**Goal proximity: Here and now versus somewhere out there in the future.** One important way to characterize goals is through the notion of the desired state’s proximity to the individual’s current state. A “proximal goal” is one where the desired state is close to the current state of individual. Goals such as “I’m going to finish this problem” and “I’m going to get an ‘A’ on this homework assignment” may indicate proximal goals because they clearly define relatively simple activities completed in a short period of time. Proximal goals make up the majority of goals in mathematics classes.

On the other hand, statements like “I want to be an astronaut” indicate a desired state that is far removed from one’s current state. These are “distal goals.” Distal goals often involve desired states centered on an ideal identity the individual wants to attain. These distal goals, if realized, necessitate that the individual create a plan—a sequence of proximal goals—and achieve that sequence of goals. While “proximal” and “distal” often refer to temporal ordering, these terms may also refer to goals in a non-
temporal way—as when they describe how close the goals are to the immediate identity of the individual.

As an example, suppose that a middle school student is asked to determine the relationship between account balance and time. Solving this problem may be considered distal as the student may have little experience with or immediate need to understand finance. To put this problem in its proper context, it may help to ask the student to define a distal goal (such as car ownership) and begin to redefine the present problem as a sub-goal that directly leads towards the (clearly) desirable outcome of owning an automobile. To assist students in seeing how it all fits together, teachers can help students develop specific proximal goals and continually encourage reflection on how the students’ efforts contribute to their long-term mathematical goals and to their long-term identity goals. Armed with a goal structure in which proximal goals can be ordered, students will tend to persevere more than when they have little clue why they are learning the material assigned to them (Bandura & Schunk, 1981).

To summarize, one dimension upon which we can classify an academic goal is its proximity to the individual. A second dimension that defines the why behind students’ perseverance behavior is the degree to which goals focus on learning content versus merely looking competent (or often, not looking incompetent). In the goal theory literature, both learning goals and performance goals fall under the umbrella of “achievement goals,” but they have important differences that we discuss in the next section. Achievement goals of both types are often framed within academics, but we emphasize that any accomplishment—academic, social, or informal—has some achievement goal attached to it.

**Goal type: Learning versus performing.** Achievement goals come in two types: “learning” (alternatively termed “mastery” goals) and “performing” (alternatively termed “ego” goals). Covington (2000) defines “learning” goals as those for which the student engages in academic tasks specifically because the student wants to increase her competency, understanding, and appreciation for the goal content. A student who struggles with difficult content because she wants to understand the concept, apply it, or use it to learn some other important concept, shows a learning goal orientation. Importantly, learning goals tend to promote perseverance because the student is satisfied (i.e., her target goal is met) only when she has truly learned the content, not when she has achieved some end performance standard regardless of whether learning has occurred.

“Performance” goals center on the outward appearance of competence in comparison with others. For example, a student rushing to finish a quiz and slamming her pencil down when finished shows a goal of appearing “quicker” than the rest of her classmates. Another student who shrinks from contributing to a class discussion because she is confused may be afraid of appearing incompetent in comparison to her peers. This student is also exhibiting a performance goal, the only difference is that this performance goal entails avoiding a bad performance rather than seeking a good one. In both examples, the students are trying to preserve their identities—the first student does so in an active manner, while the second student
does so in a passive manner focused on avoidance. Either way, learning new content is not the primary point of performance goals. In both cases, we can see that performance goals do not tend to promote perseverance.

These two goal orientations—learning versus performance—affect the way a student views a goal and will help to shape her decisions about whether to quit, persist, or persevere. An additional distinction we can clearly see in our examples of performance goals deals with “approach” and “avoidance” tendencies (Harackiewicz, Barron, Pintrich, Elliot, & Thrash, 2002; Pintrich, 2000b). The general idea behind the “approach” orientation is that the student actively tries to obtain mastery if she holds learning goals and tries to look superior if she holds performance goals. In contrast, the “avoidance” orientation is manifest when the student seeks to not develop misunderstandings (in the case of learning goals) and to not look stupid (in the case of performance goals). Approach and avoidance are useful descriptors that help to further refine the learning/performance distinction that shapes perseverance decisions, as illustrated in Table 1.

<table>
<thead>
<tr>
<th>Achievement Goal Orientation</th>
<th>Learning</th>
<th>Performance</th>
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<tbody>
<tr>
<td>Approach</td>
<td>+</td>
<td>+/-</td>
</tr>
<tr>
<td>Avoidance</td>
<td>+/-</td>
<td>-</td>
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*Note: + indicates a tendency to persevere, - indicates a tendency to not persevere.*

*Table 1. Dimensions of achievement goal orientation and associated outcomes for perseverance.*

From Table 1, we can see that learning goals are generally superior motivational orientations if we value perseverance. Seeking out mastery (learning/approach) clearly promotes continued, adaptive problem-solving behavior. Avoiding misunderstanding (learning/avoidance) minimizes perseveration and allows the student to redirect her efforts toward more productive strategies in pursuit of the end goal. In contrast, performance goals are generally inferior orientations with respect to perseverance. Performance approach goals can promote perseverance (performance/approach) if looking smart is highly valued and if the student has high ability relative to her peers. If, however, the student’s ability is not superior to her peers, any thwarting of goals (e.g., not being the fastest, losing an argument, etc.) can result in a tendency to quit (performance/avoidance). Performance avoidance goals are particularly debilitating as students actively avoid challenge to prevent any semblance of incompetence (Harackiewicz, Barron, Pintrich, Elliot, & Thrash, 2002).

**What teachers can do.** Teachers can help students develop learning goals to improve the likelihood that students will persevere in mathematics. Completing homework is a pretty vague target goal; it has not identified a *why* component. Why is the completion of homework useful or important
mathematically and personally? Helping a student redefine this goal from mere completion to a *specific learning goal* would help the student direct her behavior more productively. Because proximal goals serve as signposts on the road to the distal, identity-focused goals, a teacher could help the student reframe her goal as “I want to complete this homework so that I will better understand the concept of slope. *This will help me feel more confident about mathematics.*” This new goal explicates a *why* component, making the goal a learning goal and making the goal a stepping-stone to a new distal goal of becoming a mathematically confident and competent individual.

Teachers can begin moving students towards setting learning goals by changing what is valued by classroom norms. In particular, teachers should de-emphasize the value of performance signs (e.g., first student done, highest score, getting the correct answer, etc.) and instead place more emphasis on explaining, reasoning, and communicating ideas between students. This shift in classroom norms fosters an environment that supports learning goals rather than performance goals (Scardamalia & Bereiter, 2003).

We now turn to the kinds of resources teachers can bring to bear in the classroom to help students develop interest in mathematics, establish productive mathematical identities, and set learning goals to help them overcome challenges and persevere in mathematics.

**Resources**

There are many types of resources an individual may deploy in overcoming unexpected challenges and achieving a mathematical goal. We focus on three types of resources: (1) psychological resources, (2) extrinsic resources, and (3) identity resources. Psychological resources include a student’s current abilities and skills as well as their memories and beliefs about their ability to learn. Extrinsic resources include tools, technologies, aids that can be brought to bear on a tough problem. Identity resources include the learners’ understanding of themselves in relation to a community, and the (potential) inspiration and structure the community provides. We briefly discuss each of these three types of resources. Like each of the other critical factors in motivation and perseverance we have discussed thus far, these three types of resources play off each other as students utilize them to solve problems.

**Psychological resources.** Psychological resources are cognitive characteristics that one appraises when deciding whether to persevere. Such resources include the individual’s image of his or her self-efficacy, forbearance, aptitude, willpower, work ethic, interest, motivation, and stress management. The learner’s appraisal of her psychological resources equips her with an image of how capable she is of persevering through unanticipated challenges to achieve a particular goal. If a learner favors the psychological resources at her disposal—that is, if she considers her psychological resources sufficient to allow her to overcome the unanticipated challenges that must be overcome to achieve the goal—it is at least *possible* that she will persevere through those challenges.
In addition to appraising one’s psychological resources as sufficient to overcome unforeseen obstacles to achieve a particular goal, students must also have confidence that mobilizing their psychological resources actually will result in achieving the goal. In other words, students must view achieving the goal as being within their locus of control. For instance, although a student may consider herself competent enough to earn an “A” in her mathematics class, the student will be less likely to persevere to achieve this goal if she believes her teacher grades unfairly or assigns grades that do not accurately reflect her mathematical knowledge. In this case, the student is likely to be skeptical that utilizing her psychological resources will actually result in achievement of her goal. Persevering thus relies upon both (1) appraisal of psychological resources as sufficient for achieving mathematical goals, and (2) belief that using one’s psychological resources will result in achieving mathematical goals. Improving one’s psychological resources and developing confidence that one’s mathematical goals are attainable by utilizing psychological resources is heavily contingent upon the extent to which students attribute goal attainment to effort.

The role of effort. Psychologically, attributing one’s mathematical successes to effort and non-successes to lack of effort is a generally healthy mindset that enables students to overcome obstacles and persevere (Dweck, 2006). In particular, students who attribute their successes to internal factors, like effort, develop the psychological resources necessary to persevere to achieve a variety of mathematical goals and begin to recognize that achieving such goals is within their locus of control. Trying hard and not ending up successful, according to this mindset, is re-defined not as failure, but as not having seen the big picture. To learn quantum mechanics, people can’t just jump in with 8th- or 9th-grade mathematical skills. They must develop the requisite skills over time—i.e., they must persevere in order to eventually gain the sought-after prize, an understanding of quantum mechanics. Recognizing this and putting together a plan to achieve such a distal goal requires a basic belief that one’s ability is up to the challenge and that considerable effort must be applied over a long period of time.

Several types of effort are relevant to our discussion of perseverance in general (see Carbonaro, 2005) and of psychological resources in particular. Rule-oriented effort describes the energy students exert just complying with basic classroom norms. Procedural effort describes the energy students exert to complete assignments: reading the text and solving problems. Lastly, intellectual effort describes the ways in which a student focuses on understanding the course material: studying or doing extra work beyond assignments. This third type of effort—studying, rehearsing, and stretching beyond the class—tends to benefit long-term retention and learning more so than procedural effort does (Barnett, Sonnert, & Sadler, 2014). In fact, too much procedural effort (e.g., reading the mathematics text) has been found to be negatively associated with performance. Productive effort, in terms of mathematics performance, is reasonably synonymous with intellectual effort.
How, then, can students favorably appraise their psychological resources so that they may overcome obstacles in the way of their mathematical goals? For over 30 years, we have seen that effort (i.e., the utilization of psychological resources) depends upon a person’s expectations of success, but moreover, it also hinges on the causes to which someone attributes his or her successes and failures (Kloosterman, 1988). Research shows that people have two very different kinds of beliefs about the nature of ability: (1) ability can be fixed; or (2) ability can be malleable (Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 1996). As shown in Table 2, these beliefs about the nature of one’s ability—fixed (i.e., stable) versus malleable (i.e., unstable)—combine with beliefs about locus of control—internal versus external—to influence how a person interprets and behaves in response to successes and failures on academic tasks.

Table 2 identifies three attribution combinations (denoted by plus signs in the table) that are associated with desirable interpretations and behaviors. The first, stable, internal, success attributions, exist when learners believe the locus of control is internal (i.e., it is their ability, not external factors, that determines outcomes), when learners view ability as being stable or “fixed” (i.e., it is like a “math gene” and cannot be changed), and when learners experience successes on academic tasks. Under these conditions, the result is positive self-efficacy and interest in the domain. The second desirable scenario involves unstable, internal, success attributions. In this scenario, students experience successes and attribute those successes to ability, but rather than viewing ability as fixed, they view ability as unstable or “malleable” (i.e., it can be improved through learning). In these circumstances, the individual perceives effort to be the primary cause of success, and the natural response is to expend more effort to become more successful. The third desirable attribution scenario involves unstable, internal, nonsuccess attributions. Here students experience failure, but because they view ability as being malleable and because they view the locus of control as being internal, they attribute their lack of success in mathematics to the proper root cause: lack of effort. This implies that with improved effort, improved learning and success will result, and therefore in condition 3 (as in condition 2) the natural response is to
expend more effort to become more successful.

From this discussion, we can see that psychological resources (i.e., beliefs about one’s own abilities) matter deeply in determining students’ engagement patterns. However, though attributing one’s successes to effort is better than attributing successes to external causes, it is not necessarily enough. Productive effort—effort that focuses on studying and augmenting learning—is likely to lead to greater success than mere compliance on assignments, no matter how challenging those assignments may be. The research on productive effort suggests that among low-performing students, careful assessment of students’ study behaviors and their productivity may need to be undertaken before telling them to just expend more effort and “try harder” (Barnett, Sonnert, & Sadler 2014).

**Extrinsic resources.** Extrinsic resources are tangible resources that an individual considers useful for overcoming the unexpected challenges she encounters in pursuit of a particular goal. Examples of extrinsic resources include time, materials, and help and support from other people. Even if the student appraises her psychological resources as insufficient to overcome certain obstacles, she may persist in pursuing a goal if she appraises the combination of psychological and extrinsic resources to be sufficient. In the context of mathematics, common extrinsic resources include computational tools, computer algebra systems, manipulatives, representational tools, and dynamic visualization software. Such extrinsic resources allow students access to powerful ideas and help them structure their thinking more effectively than they could have otherwise.

**Identity resources.** A student may also utilize identity resources to assist in overcoming unexpected obstacles to achieve a goal. Sometimes failing to achieve a particular goal results in failing to achieve or maintain a particular type of identity. In other words, while pursuing a goal, an individual may think, “I want to achieve this goal because it will allow me to be the kind of person I want to be.” But often an individual also anticipates the identity that he or she would have in the event of failure to achieve the goal: “What kind of person will I be if I do not achieve this goal? What kind of community will I be (or not be) a member of? How will others perceive me?” The answers to these questions can serve as motivation and thus encourage someone to persevere. In other words, the identity consequences of success and failure can provide an additional incentive to overcome the unanticipated challenges and obstacles encountered in pursuit of one’s goals. In this way, identity resources play a role in determining one’s ability to persevere.

**Stereotype threat.** Identity resources are not immune to social factors. In particular, stereotype threat may negatively (or positively) influence the extent to which a student is able to productively exploit identity resources in the process of persevering to achieve mathematical goals. Someone who believes oneself to be a member of a group that has traditionally performed poorly compared to others may internalize an ability-related belief associated with underperformance. For example, when women are told
that women tend to do more poorly with particular content, they tend to perform more poorly than men on tests of that content, even when item difficulty is controlled for (Spencer, Steele, & Quinn, 1999). The interpretation of these findings is that when people are given reason to believe that difficult tasks tend to be out of the ability of a reference group to which they belong, they tend to internalize this “stereotype threat” and perform more poorly than they would have had they not been given the information. Moreover, if stereotype threat is decreased by informing women that there are no differences in men’s or women’s performance on test items, they tend to do better than when they are given no information, thus indicating that stereotype threat exists in the back of their minds even when they are not explicitly reminded of it. Such beliefs may be a critical causal factor for dropping mathematics as an interest and as a basis of self-evaluation. These findings have been shown to generalize across ethnicity and gender. Even white males, when told that Asians tend to do better at mathematics tasks, tend to perform worse than they would have without the reminder (Aronson et al., 1999). Additionally, and perhaps more detrimentally, the individual will not consider failing to achieve mathematical goals as a threat to his or her identity since doing so is, from the students’ perspective, consistent with popular opinion. It is in this way that stereotype threats maintain the potential to negate the utility of other, positive identity resources a student may possess.

Steele, Spencer, and Aronson (2002) make a critical distinction between disengagement in a domain (mathematics) versus disidentification with the domain. Disengagement corresponds to a short-term lack of persistence (and therefore lack of perseverance) due to the perceived fit of the task to the perceived abilities and nascent identity of the learner. Disidentification corresponds to the gradual whittling away of value for the domain, and even resistance in developing a personal connection with the domain. As with the development of interests, the development of mathematical identity is dependent upon continual, positive experiences that effectively employ psychological resources in pursuit of mathematical goals (Woodcock, Hernandez, Estrada, & Schultz, 2012).

What teachers can do. The extent to which an individual perseveres is determined in part by his or her appraisal of resources available to overcome unforeseen challenges in pursuit of a goal. It follows that teachers can influence students’ perseverance by helping students appraise their available resources. The remainder of this section outlines various strategies that teachers may employ to support students in persevering through mathematical challenges by allowing them to favorably appraise the psychological, extrinsic, and identity resources at their disposal.

To encourage students to develop the psychological and identity resources necessary to persevere to achieve mathematical goals, students should first recognize mathematical proficiency as something worth persevering for. Teachers can and should allow students to redefine what it means to participate in the act of doing mathematics and allow students to redefine what it means to be proficient at mathematics
so that students may recognize mathematical proficiency as something worth persevering for. Students will then be equipped to make explicit goals that lead towards mathematical proficiency. In this way, teachers can help students recognize mathematical knowledge as a component of a desired or sought identity.

When students recognize mathematical proficiency as an aspect of a desired or sought identity, they can anticipate how they would feel about the identity consequences of achieving or failing to achieve particular mathematical goals and use that anticipation as motivation to persevere through unforeseen challenges. In this way, students may construct an identity resource that contributes to their persevering to achieve mathematical goals.

Once students perceive mathematics as something worth knowing that has some identity stock, then they need to enhance their psychological resources by becoming confident in their own mathematical abilities. For this to happen, teachers should help students redefine the skill set required to achieve mathematical proficiency and help them recognize that reasoning, creativity, and innovation are characteristics necessary for mathematical proficiency, as opposed to simply memorization, rule-following, and so forth. Moreover, teachers should help students recognize that they possess these desirable characteristics. It is important to note that teachers cannot just tell students that these characteristics are required for success in mathematics. Students have to experience the utility of these competencies in mathematical contexts. Teachers should therefore place students in situations that require the use of desirable characteristics such as reasoning, creativity, and innovation so that students may experience the efficacy of qualities. When this happens, students will come to appreciate their psychological resources because they have experienced how they can be used to allow them to achieve their mathematical goals.

Finally, teachers should support students in persevering to achieve mathematical goals by making a variety of appropriate extrinsic resources available to students. Such extrinsic resources include computational and conceptual tools like calculators, computer algebra systems, physical manipulatives, and dynamic visualization software. Students need to have access to these types of resources and understand how to use them to pursue mathematics-learning goals. Having access to appropriate computational and conceptual tools allows students to apply their psychological resources to the aspects of the goal pursuit that demand them most. That is, thoughtful use of these extrinsic resources reduces the cognitive demand needed to achieve a mathematics-learning goal by allowing students to apply psychological resources more economically.

In addition to material tools, extrinsic resources include the psychological resources of other individuals—namely, those of other students. If teachers can engage groups of students in developmentally appropriate tasks while holding the students intellectually responsible for each other,
then students are encouraged to recruit the psychological resources of their peers to succeed in school. In the process, any particular student may come to understand the intellectual strengths and weaknesses of her classmates and get a sense of how the psychological resources of others may assist her in achieving her mathematics learning goals.

**Consequences**

People tend to enjoy and persevere with activities in which they are successful, and they tend to avoid activities in which they are unsuccessful. But how do learners define success, and to what extent is success in mathematics valued when compared to other pursuits? In this section, we address this question by examining tasks that are intrinsically motivating versus those that are extrinsically motivating. In general, we can see that intrinsic motivation leads students to pursue challenging content and persevere through difficulties, whereas extrinsic motivation is effective only to induce someone to do something they wouldn’t ordinarily do without some reward or punishment hanging over their head. Finally, we discuss the perceived usefulness of mathematics as a necessary condition for the development of intrinsic motivation and situational interest.

**Intrinsic versus extrinsic motivation.** Rewards and punishments serve as incentives for behavior, but not all consequences of tasks result in tangible reinforcements. Some activities seem to be entered into without regard to their payoff. Motivation researchers have termed such motivation *intrinsic* (Ryan & Deci, 2000). An example of intrinsic motivation would be a student who enjoys mathematics because she views mathematical activities as fun, logical puzzles. For this student, completing a task for a grade may only factor into her engagement secondarily. The vast majority of research shows that when students’ primary motivation is intrinsic—where the salient consequences are fun, pleasure, and the pursuit of interest—they tend to persist in the face of failure, show more creative problem-solving behaviors, and value the domain, more than students who are extrinsically motivated.

Students who are *extrinsically* motivated, on the other hand, may work diligently to obtain a reward they value or to escape some consequence they deem sufficiently negative (Simons, Dewitte, & Lens, 2000). The most common example of extrinsically motivated behaviors concerns the grading system. Because nearly every task a student engages in has a grade, and because these grades are associated with real consequences (e.g., social recognition, enhanced academic experiences) and the affective feelings they engender (satisfaction versus shame, for example), grades are powerful predictors of persistent behavior. Research shows that grades can work to decrease intrinsic motivation and interfere with the process and quality of learning, often by distracting the learner from the learning process to focus only on the outcome (Docan, 2006; Deci & Ryan, 1987).

**Utility and its impact.** We teach mathematics in schools and argue for increased rigor in mathematics not because it has inherent intrinsic value, but primarily because of its potential utility. This
utility has a powerful influence on the perceived worth of engagement in mathematics and on the maintenance of effort over time in math. Perceived instrumentality is a term that researchers use to describe how useful a student views a task in terms of its ability to help her achieve her goals (Husman, Derryberry, Crowson, & Lomax, 2004). For example, a student with the goal of becoming a personal trainer might view a trigonometry task as having little impact on her career and would thus assign the task little perceived instrumentality. In contrast, a student intent on becoming an engineer may view the same trigonometric task as instrumental to becoming an engineer. In both cases, the student assesses the mathematical task relative to her goals to determine the task’s instrumentality (Hilpert et al., 2012).

Perceived instrumentality is governed by two factors: immediate utility and future utility. Immediate utility helps determine the value the student places on a task. For example, learning linear functions for the purpose of determining appropriate weight loss regimens has immediate utility to the personal trainer, who will attend to this content more (and persevere with it to a greater extent) in order to be able to apply the content. Most mathematics taught in school, however, is not immediately useful—think of how immediately useful trigonometry is in the life of an 8th grader. When teaching math, we often appeal to the future utility of the content, hoping that the delayed gratification of college, jobs, and money will compel students to apply effort in mathematics. Students who do consider the task’s instrumentality within the context of future aspirations and goals are more likely to have enhanced motivation, performance, and persistence (Simons, 2000). This translates to the students being more likely to persevere over the long haul.

What teachers can do. One way a teacher can help motivate her students is by enhancing the perceived instrumentality of mathematics tasks and activities (Creten, Lens, & Simons, 2001). Polling students’ career aspirations enables teachers to choose tasks that directly pertain to students’ future goals. In addition, simply asking students to contemplate the utility of their mathematical experiences has been shown to impact student motivation. Teachers who ask a question like, “How might this concept be useful for what you want to do after you graduate and go to work?” can get students thinking about the utility of the mathematics they’re learning (Husman et al., 2004).

In a basic sense, connecting mathematics to students’ interests improves students’ sense of intrinsic motivation. Teachers should do what they can to illustrate to students that the mathematics they are studying has both immediate and future value. Algebra, for example, is a critical tool for future mathematics courses, but this isn’t particularly compelling to the 8th grader. Instead, providing mathematics tasks that relate to student activities—for example, planning end of year events, decorating for parties, sports performance, and other potential individual interests—shows students how the algebra is a way of thinking and can be applied in these student activities. Students can be shown how to use what they learn in mathematics to maximize the number of people involved, minimize cost, increase
performance, and reduce hassle, which are all potentially important goals. Tying tasks to individual interests like aviation, astronomy, art and design, can also help students see how valuable algebra is for their nascent identities to come to fruition.

**Conclusion**

In this paper, we have integrated a number of related lines of inquiry in the field of mathematics motivation. We have argued that perseverance is not a fixed and immutable psychological trait, but rather that perseverance is determined by the interaction of several anticipatory variables, including students’ interests, goals, and identity, in response to challenge. Interest directs students’ behaviors by enabling them to compare current requirements for participation in mathematics tasks with their stored memories of past situational interests. The goals students construct also direct behavior by organizing potential behaviors into longer-term strategies that lead to a desired future self and identity. Learning goals enable students to persevere because non-successes are not interpreted as failures and can be overcome with effort and the development of new strategies with new proximal subgoals. Patterns of successes and failures students experience in mathematics help structure their mathematical identity—their place in the mathematics community and their personal feelings of worth in relation to their mathematical experiences. Interest, goals, and identity are used to anticipate the potential value that engagement in mathematical pursuits might hold for the student, and they help determine at what point students’ involvement in mathematics becomes sub-optimal.

As students choose to engage in mathematics tasks, two additional factors come into play: the perceived utility of the content and the resources students can bring to bear on the problem to enhance their chances of success. The more useful students find the mathematics they are learning, the more situational interest they tend to feel and the more effort they are willing to expend (Middleton, 2013). If this utility is focused on learning material that is central to students’ personal goals, such as becoming competent or succeeding in a particular career, students tend to develop intrinsic motivation for the content itself. But if completion of a mathematical task is only valuable in service of obtaining a grade or other reward, or for escaping some sanction placed on poor performance, the student will tend to value the outcomes and not the content, exhibiting extrinsic motivation.

Together these variables influence the amount of sustained effort a student is willing to put forth on mathematics and mathematics-related activities. Research has shown that effort is the great equalizer (Chouinard, Karsenti, & Roy, 2007). It may seem obvious, but the more productive effort a person expends in an academic endeavor, the more he or she will accomplish! The important point here is that if mathematical tasks (1) tap into students’ interests; (2) incorporate explicit learning goals; (3) support a robust mathematical identity; (4) emphasize the usefulness of mathematics for supporting the students’
own goals; and (5) recruit social, material, and intellectual resources to improve problem solving, students will put forth more effort, show more cognitive efficiency, and enjoy mathematics more than is currently the norm across the nation in K-12 classrooms.

Since students’ mathematical experiences are largely standardized over their 13 years in K-12 schooling (and beyond), students do develop patterns of engagement for mathematics that are predictable individually and collectively. For US students, these patterns are not yet aligned towards a high degree of mathematical perseverance—most students just get by or leave mathematics once they accomplish the minimum requirements. To enhance grit, hardiness, gumption or any other name we could place on perseverance, we must first make the mathematical curriculum more challenging, but we must also enhance the potential for student buy-in by incorporating the principles that, in this paper, we have identified as fundamental to continued value of and engagement in mathematics.
References


